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Learning Direct and Inverse Proportion: A Study in the Framework of Pedagogical Content Knowledge

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Abstract

Many real-life problems are closely related to the concepts of direct and inverse proportion. Therefore, it is essential for students to develop a sound understanding of these concepts. This study aims to observe the mathematics learning process in the classroom, specifically in relation to pedagogical and content knowledge regarding direct and inverse proportion. This research employed a qualitative observational approach to examine how the teaching and learning process unfolds in the context of these mathematical concepts. The study took place on November 28, 2024, involving 28 seventh-grade students from a private junior high school in Bandung, Indonesia. Data were collected through direct classroom observation during the lesson. The primary research instrument was an observation sheet, which was developed to examine the implementation of Pedagogical Content Knowledge (PCK) in mathematics instruction. The data analysis process consisted of data processing, data analysis, and data interpretation. The observation of seventh-grade mathematics instruction focusing on the concepts of direct and inverse proportion from a PCK perspective provides insight into how PCK is applied in the mathematics classroom. The predominant use of lecture-based teaching limited students' opportunities to construct conceptual understanding, resulting in noticeable difficulties in distinguishing between direct and inverse proportion problems. It was found that many students lacked a clear understanding of the differences between direct and inverse proportions, leading to difficulties in identifying whether a given problem involved one or the other. This indicates a need for instructional methods that allow students to construct their own understanding of the topic being taught. Furthermore, the development of instructional materials should be more closely aligned with students' cognitive characteristics to support meaningful learning experiences.

Keywords: Direct and Inverse Proportion; Content Knowledge (CK) and Pedagogical Content Knowledge (PCK)

Abstrak

Permasalahan dalam kehidupan sehari-hari banyak sekali berkaitan dengan konsep perbandingan senilai dan berbalik nilai, oleh sebab itu perlu bagi siswa untuk memahami konsep ini. Penelitian ini bertujuan untuk mengamati proses pembelajaran matematika di dalam kelas yang ditinjau dari pedagogik dan pengetahuan tentang konten perbandingan senilai dan berbalik nilai. penelitian ini merupakan studi kualitatif observasional, metode ini digunakan untuk menyelidiki proses pembelajaran di kelas pada materi perbandingan senilai dan berbalik nilai. Penelitian ini dilaksanakan pada tanggal 28 November 2024, dengan partisipan yaitu 28 orang siswa kelas VII pada salah satu SMP swasta di Kota Bandung. Teknik pengumpulan data pada penelitian ini adalah dengan melakukan pengamatan langsung pada saat proses pembelajaran. Instrumen yang digunakan pada penelitian ini berupa lembar observasi yang dikembangkan untuk mengamati PCK pada pembelajaran di kelas. Teknik analisis data pada penelitian ini terdiri atas pemrosesan data, analisis data, dan interpretasi data Pembelajaran matematika kelas VII dengan konsep perbandingan senilai dan berbalik nilai yang ditinjau dari sudut pandang PCK memberikan gambaran tentang bagaimana PCK diterapkan dalam pembelajaran matematika di dalam kelas. Metode ceramah yang digunakan guru menimbulkan dampak yang membuat siswa kurang memaknai apa yang sedang dipelajari. Kurangnya pemahaman siswa tentang perbedaan antara konsep dari perbandingan senilai dan berbalik nilai yang mengakibatkan siswa tidak bisa membedakan permasalahan yang sedang dibahas apakah permasalahan perbandingan senilai atau perbandingan berbalik nilai. Diperlukan metode pembelajaran yang dapat membuat siswa membangun pengetahuannya sendiri tentang materi yang sedang dipelajari, Selain itu, pengembangan bahan ajar melalui proses penelusuran dan pengkajian yang lebih mendalam dan telah disesuaikan dengan karakteristik siswa, sehingga pada akhirnya siswa dapat memaknai apa yang sedang dipelajarinya.

Kata Kunci: Perbandingan Senilai dan Berbalik Nilai; Content Knowledge (CK); Pedagogical Content Knowledge (PCK)

Introduction

The concepts of ratio and proportion are fundamental topics in mathematics education, particularly at the secondary school students (Lo, Watanabe, & Cai, 2004). In particular, a solid understanding of direct and inverse proportion is essential for students, as these concepts are closely connected to a wide range of real-world problems. Therefore, it is essential for students to develop a solid understanding of both types of proportion. Research has shown that students who comprehend the concepts of direct and inverse proportion tend to be better prepared and more capable of solving complex problems, whether in mathematics or in daily life contexts (Agnesti & Amelia, 2021). Conversely, several studies have also revealed that many students face difficulties in understanding these concepts, which hinders their ability to solve related problems (Angraerni, 2021; Kartiko & Mampouw, 2021; Setyaningrum & Mampouw, 2020).

One of the key factors contributing to students' difficulties is the limited effectiveness of the instructional approaches employed in the classroom. Teaching methods that emphasize rote memorization over conceptual understanding often lead to errors in applying direct and inverse proportion (Sartika, Anim, & Sitorus, 2024; Nugraha, Sujadi, & Pangadi, 2016). Moreover, the teacher's mastery of the subject matter cannot be overlooked. Loewenberg Ball, Thames, & Phelps (2008) argue that teachers are expected to have a strong command of the subjects they teach. A lack of subject matter knowledge can limit a teacher's ability to assist students in understanding mathematical content. Therefore, teachers must adopt more innovative and creative approaches, utilizing contextual and interactive teaching methods. These strategies have been shown to enhance students' understanding and engagement with the concepts of direct and inverse proportion (Rahmasantika & Prahmana, 2019; Hamidah, Putri, & Somakim, 2018).

Effective instruction tailored to students' characteristics requires a combination of content knowledge and pedagogy, commonly referred to as Pedagogical Content Knowledge (PCK). The distinction between PCK and content knowledge (CK) has been widely acknowledged by the educational research community (Krauss et al., 2008). Content knowledge (CK) refers to the concepts, principles, relationships, processes, and applications that students are expected to learn in a given subject area, aligned with their cognitive development and knowledge organization (Özden, 2008). The interrelation between teachers' pedagogical beliefs and students' understanding of pedagogical content has also been highlighted in the literature (Peterson, Fennema, Carpenter, & Loef, 1989). According to Gudmundsdottir & Shulman (1987), PCK includes knowledge of the most effective ways to represent and explain subject matter in a way that is accessible to learners, as well as an understanding of students' conceptions and misconceptions. A deep understanding of PCK enables teachers to design meaningful and student-centered instructional strategies (Barut, Wijaya, & Retnawati, 2021; Rahayu, Muhtadi, & Ridwan, 2022).

With strong PCK, teachers can adapt their teaching methods to suit student needs—for example, by using problem-based learning (PBL) or contextual learning approaches that are directly relevant to students' everyday lives (Setyaningrum & Mampouw, 2020). When combined with appropriate instructional models such as PBL, PCK contributes significantly to the development of students' critical and creative thinking skills (Marchelin, Hamidah, & Resti, 2022). Moreover, Restiana &

Pujiastuti (2019) found that the components of Technological Knowledge (TK) and Technological Content Knowledge (TCK) have a significant positive impact on teachers' Technological Pedagogical Content Knowledge (TPACK). However, this study focuses exclusively on teachers' Pedagogical Content Knowledge (PCK), with technological aspects not being the primary concern. This is because the research emphasizes the pedagogical and content dimensions of teaching, rather than the integration of technology. In conclusion, PCK plays a pivotal role in teaching the concepts of direct and inverse proportion. Through a comprehensive understanding of PCK, teachers can design more effective learning experiences, support students in overcoming learning challenges, and foster critical thinking skills essential for success in mathematics and beyond (Rahayu et al., 2022; Rahmasantika & Prahmana, 2019).

Method

The method employed in this study is a qualitative observational approach to investigate the classroom learning process on the topic of direct and inverse proportions. The research was conducted on November 28, 2024, involving 28 seventh-grade students from a private junior high school in Bandung, Indonesia. Data collection was carried out through direct observation during the teaching and learning process. The primary instrument used in this study was an observation sheet, developed to observe Pedagogical Content Knowledge (PCK) during classroom instruction. The purpose of this observation was to examine how the teacher delivered the lesson, as well as to observe how students received and responded to the instruction. The data analysis process in this study consisted of data processing, data analysis, and data interpretation.

Results

The mathematics lesson observed on November 28, 2024, in a seventh-grade classroom at a private junior high school in Bandung consisted of three instructional periods. The lesson was organized into two sessions: the first session included two periods from 8:40 to 9:50 a.m., and the second session consisted of one period at 11:15 a.m.

The first session began with the teacher instructing students to begin the class with a prayer. One of the students led the prayer, which was followed by a collective greeting to the teacher from the class.

The teacher initiated the lesson by prompting students to recall previously learned material. Following this, the teacher asked students to read the new topic to be studied—direct and inverse proportion. The students read the material from their student worksheets (LKS), which had been distributed individually. These worksheets were published by a third-party publisher, and the sequence of topics in the lesson followed the structure provided in the LKS.

Subsequently, the teacher introduced a problem related to direct proportion. The problem was read aloud by the teacher while students were instructed to write it down. The context of the problem involved determining the price of a certain item. After presenting the problem, the teacher proceeded to guide students through the process of finding a solution.

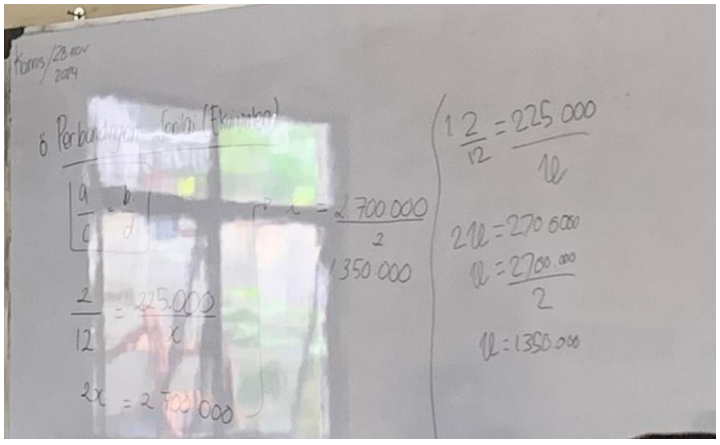


Figure 1. The teacher demonstrates the solution to the given problem.

The Solution Presented by the Teacher :

$$\frac{a}{c} = \frac{b}{d}$$

$$\frac{2}{12} = \frac{225.000}{x}$$

$$2x = 2.700.000$$

$$x = \frac{2.700.000}{2}$$

$$x = 1.350.000$$

As shown in Figure 1, the solution presented by the teacher utilized an algebraic approach, specifically through the use of a linear equation. This method caused some confusion among students, as they had not yet been introduced to the concept of linear equations. Furthermore, the students were unfamiliar with the meaning of the variable x used in the solution process. As a result, they struggled to comprehend the reasoning behind the steps taken to solve the given problem.


Semester Ganjil

Daftar Isi

Daftar Isi 1
Bab 1 Bilangan Bulat 2
A. Memahami Bilangan Bulat 3
B. Operasi Hitung Bilangan Bulat 5
C. Faktor Bilangan Bulat 10
Uji Kompetensi 15
Bab 2 Bilangan Rasional 20
A. Di Antara Dua Bilangan Bulat 21
B. Membandingkan Bilangan Rasional 24
C. Operasi Hitung Bilangan Rasional 27
Uji Kompetensi 38
Ulangan Tengah Semester 46
Bab 3 Rasio 50
A. Konsep Rasio 51
B. Skala 52
C. Laju Perubahan Satuan 53
Uji Kompetensi 55
Ulangan Akhir Semester 61
Daftar Pustaka 64

Figure 2. Curriculum structure for the first semester.

In the mathematics subject for the odd semester, seventh-grade students at this school studied three main topics, which included integers, rational numbers, and ratios, as shown in Figure 2.

After explaining the solution method for problems related to direct proportion, the teacher proceeded to give students a set of practice problems focused on the concept. As in the previous activity, the teacher read the problem aloud while students were instructed to write it down in their exercise books. The practice problem provided by the teacher was related to the relationship between fuel consumption and travel distance—an example of a direct proportion context. Following the presentation of the problem, the teacher allowed students time to work independently to find a solution. Students who completed the task were then encouraged to confirm their answers with the teacher.



Figure 3. Students working on the problem given by the teacher.

Five students who completed the task correctly were given appreciation by having a sticker placed on their exercise books, as shown in Figure 4.



Figure 4. Sticker on the student's exercise book.

The teacher then invited a student who had provided the correct answer to present the solution on the whiteboard in front of the class. Afterward, the teacher provided a brief explanation of the student's solution before the first lesson ended.

The second lesson began with a brief review of the concept of direct proportion that had been covered earlier. The teacher then introduced a problem related to inverse proportion, which involved the price of red onions. The teacher asked the students whether the given problem represented a direct proportion or an inverse proportion. Based on their previous learning experience, the students provided a variety of responses, with some stating that the problem was an example of direct proportion, while others claimed it was an example of inverse proportion, as shown in Figure 5.



Figure 5. Students providing arguments about the given problem.

Similar to the first lesson, the teacher presented a problem by reading it aloud and asked the students to write the problem in their exercise books. The students were then instructed to solve the problem. Once they had finished, they were asked to confirm their answers with the teacher. The first five students to confirm their correct answers were rewarded with a sticker placed on their exercise books as a form of appreciation. Next, one student was asked to write the correct solution on the whiteboard, followed by the teacher's explanation of the student's answer, as shown in Figure 6. Before the lesson ended, the teacher assigned homework (HW) for the students to complete at home. The homework involved solving several problems in the LKS (student worksheets), which would be collected in the next class meeting.

Discussion

The teaching of direct proportion in the seventh-grade class was carried out using a lecture method by the teacher. Although the lesson was delivered using the lecture method, the teacher began by presenting problems that were closely related to the students' daily lives. This demonstrates that the teacher possesses knowledge of the context relevant to the material being taught. The presentation of contextual

problems and practice exercises plays a critical role in mathematics instruction and contributes significantly to the time allocated for mathematics lessons (Christiansen & Walther, 1986). The selection of appropriate problems and tasks, when well-implemented, provides a solid foundation for students to construct knowledge and creates substantial learning opportunities (Williams, 2002). Knowledge about the potential of presenting mathematical problems and tasks for learning is the first essential aspect of Pedagogical Content Knowledge (PCK) (Krauss et al., 2008). In line with this, Maryono (2020)found that Pedagogical Content Knowledge (PCK) significantly contributes to teachers' effectiveness in teaching mathematics in the classroom. Additionally, the study indicated that teachers' educational background also affects their proficiency in content knowledge.

After presenting contextual problems, the teacher proceeded to teach students how to find solutions to the given problems. The solution presented by the teacher was based on the concept of linear equations, specifically the concept of variables. Although the teacher had a sufficient understanding of the concept being taught, the solution caused initial confusion among students due to the use of linear equation concepts, which were not previously covered. This confusion arose because the teacher followed the sequence of materials in the Student Worksheet (LKS), which was not fully aligned with the students' prior knowledge. As a result, students resorted to simply mimicking the solution procedure demonstrated by the teacher. When asked about the meaning of the variable "x" used in the solution, students were unable to explain its significance. Understanding the meaning of variables is crucial for solving problems involving linear equations (Maudy, Suryadi, & Mulyana, 2017).

Furthermore, the students' lack of understanding of the distinction between the concepts of direct and inverse proportion led to their inability to distinguish between problems related to direct proportion and those related to inverse proportion. A thorough review of the sequence of teaching materials is crucial to prevent learning barriers related to the material sequence. This aligns with the Merdeka Curriculum, which empowers teachers to organize the sequence of topics according to their students' characteristics. Although the Merdeka Curriculum was in place at the time of the study, its implementation was not fully optimized by the teacher, leading to suboptimal sequencing of topics in this odd semester.

Conclusion

The instruction of direct and inverse proportion concepts in seventh-grade mathematics, analyzed through the lens of Pedagogical Content Knowledge (PCK), offers valuable insights into how PCK is implemented in the classroom. The use of a lecture-based approach supports the teacher's ability to deliver content efficiently; however, it also contributes to students' superficial understanding of the material. The students' limited grasp of the distinctions between direct and inverse proportion hindered their ability to accurately identify the type of proportionality involved in a given problem. To address this, instructional strategies that promote student-centered learning and encourage the construction of conceptual understanding are necessary. Additionally, the development of instructional materials should involve deeper exploration and be tailored to the learners' characteristics, enabling students to better comprehend and internalize the mathematical concepts being taught.

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